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THE METHOD OF HEAT POLYNOMIALS AND SPECIAL FUNCTIONS FOR THE PROBLEM OF HEAT EQUATION IN REGIONS WITH FREE BOUNDARIES AND THEIR APPLICATION

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy (PhD) in the specialty "6D070500 - Mathematical and Computer Modeling"

The dissertation has a twofold purpose: to introduce the heat polynomials and special functions and their applications for some more refined models of (first-order) phase transitions, and also to illustrate solution method for the analysis of associated nonlinear initial- and boundary-value problems. These two aspects might hardly be separated, for the interplay between modelling and analysis is the blood and life of research on Stefan-type problems.

The main idea of using the heat polynomial method to solve applied heat transfer problems in areas with free boundaries is that this method allows you to construct a solution to the problem in a form that exactly satisfies the differential equation, and the unknown coefficients in the solution structure and expansion coefficients of the free boundary are selected in such a way as to exactly or approximately satisfy the initial and boundary conditions. This method makes it possible to obtain an approximate solution with any degree of accuracy and estimate the approximation error using the maximum principle.

The aim of the dissertation. The main goal of the dissertation is to develop new accurate and approximate analytical methods for solving heat and mass transfer problems with phase transformations of matter based on the methods of heat polynomials and their use to study the dynamics and calculation of erosion of electrical contact systems of low-voltage devices. The solution of these problems and their analysis will allow us to find new promising areas for the creation of modern low-voltage electric devices and choose the optimal modes of their switching ability. and also develop on this basis practical recommendations for reducing bridge and arc erosion of contacts, increasing the reliability and service life of electrical devices, saving precious and scarce electrical contact materials (silver, gold, platinum, tungsten) for equivalent composite materials.

Dissertation objectives. The dissertation consists of the following tasks:

- One-dimensional heat polynomials and associated functions;
- Heat polynomials for solving spherical and cylindrical heat conduction problems;
- Special functions for solving heat process problems in electrical contact processes.
- Similarity solution of heat problems with temperature dependence coefficients.

To build mathematical models describing the processes of heat and mass transfer, provided for in the first task of the dissertation, you must first systematize the known properties of one-dimensional heat polynomials and supplement them with new properties that will make it possible to use them to solve problems in areas with free boundaries and estimate the approximate error solutions. The solution of spherical problems will be implemented using their reduction to the corresponding plane-one-dimensional problems. It is supposed to solve cylindrical problems using the connection of heat polynomials with Laguerre polynomials. Using the obtained solution, a description of the dynamics of the free boundary will be given and a procedure will be developed for calculating the motion of phase transition isotherms (softening, melting, and evaporation) of the electrode material.

For the generalized heat equation, the corresponding heat polynomials and associated functions will be constructed, for which it is supposed to find the generating function and use the Appel transform. Using the constructed heat polynomials, a solution to the problem of coupling the spherical and axial models by the method of heat polynomials will be obtained and a method for calculating the dynamics of the bridge and the bridge erosion of electrical contacts will be developed.

Scientific novelty and significance of the dissertation. Recently, analytical methods for solving heat and mass transfer problems have received a new incentive for their further development due to the growing need to solve multicriteria problems for which numerical methods are unable to assess the influence of a large number of input parameters on the behavior of the solution and especially on its dynamics. In particular, to solve the Stefan type problems with a free boundary that describe heat transfer with phase transitions, the integral method of heat balance [1] - [3], the perturbation method [4] - [6], and a number of other methods are widely used. The main problem when using this method is the estimation of the approximation error, which for applied problems, as a rule, is replaced by a comparison of the analytical solution with experimental data. In contrast to this, the method of heat polynomials developed in this dissertation makes it possible to precisely satisfy the differential heat equation, and the error in satisfying the initial and boundary conditions can be estimated using the maximum principle. Heat polynomials introduced by P.S. Rosenbloom and D.V. Widder, can be considered as basic functions for constructing a solution to the heat equation in the form of their linear combination. A number of interesting results have been obtained in this area for solving the classical problems of heat conduction.

One of the important areas of application of problems with a free boundary is the mathematical modeling of phenomena in the low-temperature plasma of an electric arc and the contacts of electrical devices. An analysis of the solutions makes it possible to test the theoretical results obtained, verify the effectiveness of the developed algorithms for specific evolutionary processes in electrical devices, and give an interpretation of the available experimental data.

The method of heat polynomials will be used to solve spherical and cylindrical problems with phase transitions (softening and melting) that arise when studying the process of heating closed electrical contacts in magnetic fields, which will determine the limiting welding currents. To solve heat transfer problems with phase transitions in bodies with a variable cross-section (liquid metal bridge, electric arc), an apparatus of heat polynomials will be developed that generalizes one-dimensional heat polynomials, for which a generating function will be found and the corresponding associated functions biorthogonal to generalized heat functions will be constructed using the Appel transform polynomials.

The method of research. For spherical one and two-phase Stefan problem heat polynomials and special function (integral error function) will be used for solution. For generalized heat problems special function method (Laguerre polynomials and congruent hypergeometric functions) will be considered. In spherical Stefan problem radial heat polynomials are also effective. To approximate problems using heat polynomials there will be used variational and collocation approximation methods. Convergence of series represented by linear combination of heat polynomials and special functions will be proved.

The similarity method will be used to solve spherical Stefan problem with temperature dependence coefficients. This method is very useful to reduce Stefan problem with partial differential equation to ordinary second order nonlinear differential equation, it is helpful to simplify the problem and solving it by using integral equation Volterra type. Existence of the solution of nonlinear model will be proved by using fixed point theorem.

Publications. On the topic of the dissertation 11 papers were published and accepted: 7 journal articles (3 of them in Scopus databases journals with quartile Q3, 4 in journals recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan, also 3 of them indexed in Clarivate Web of Science), and 4 works in collections of international scientific conferences (one of them indexed in Scopus).

The results on the topic of the dissertation were published in the following journals:

Publication in the journal included in the international base Scopus or Web of Science

1. T.A. Nauryz. Existence and uniqueness for one-phase spherical Stefan problem with nonlinear thermal coefficients and heat flux condition. International Journal of Applied Mathematics, 2022, 35(5), pp. 645-659. **(Scopus, SJR 0.27, percentile 40)**, DOI: http://dx.doi.org/10.12732/ijam.v35i5.2

2. S.N. Kharin, T.A. Nauryz. Solution of two-phase cylindrical direct Stefan problem by using special functions in electrical contact processes. International Journal of Applied Mathematics, 2021, 34(2), pp 237-248. **(Scopus, SJR 0.268, percentile 31)**, DOI: http://dx.doi.org/10.12732/ijam.v34i2.

3. S.N. Kharin, T.A. Nauryz. One-phase spherical Stefan problem with temperature dependent coefficients. Eurasian Mathematical Journal, 2021, 12(1), pp 49-56. **(Scopus, SJR 0.277, percentile 25)**, DOI: https://doi.org/10.32523/2077-9879-2021-12-1-49-56.

CQASES

1. S.N. Kharin, T.A. Nauryz, B. Miedzinski. Two phase spherical Stefan inverse problem solution with linear combination of radial heat polynomials and integral error functions in electrical contact process. International Journal of Mathematics and Physics**,** Vol 11, No. 2, (2020), pp. 4-13, **(CQASES, Clarivate Web of Science),** DOI: https://doi.org/10.26577/ijmph.2020.v11.i2.01

2. S.N. Kharin, T.A. Nauryz. Two-phase Stefan problem for generalized heat equation. News of the National Academy of Sciences of the Republic of Kazakhstan, Vol. 2, No. 330 (2020), pp. 40-49. **(CQASES, Clarivate Web of Science)**, DOI: https://doi.org/10.32014/2020.2518-1726.13.

3. S.N. Kharin, T.A. Nauryz. Two-phase spherical Stefan problem with non-linear thermal conductivity. Kazakh Mathematical Journal, 20:1 (2020), pp.27-37. **(CQASES)**.

4. S.N. Kharin, T.A. Nauryz, K. Jabbarkhanov. Solving two-phase spherical Stefan problem using heat polynomials. Kazakh Mathematical Journal, 19:1 (2019), pp. 59-68. **(CQASES)**.

Publications in materials of international conferences

1. S.N. Kharin, T.A. Nauryz. Mathematical model of a short arc at the blow-off repulsion of electrical contacts during the transition from metallic phase to gaseous phase. Fifth International Conference on Analysis and Applied Mathematics, Turkey, AIP Conference Proceedings 2325, 020007 (2021); **(Scopus, percentile 17).** DOI: https://doi.org/10.1063/5.0040412.

2. S.N. Kharin, T.A. Nauryz. Stefan's problem with nonlinear thermophysical characteristics. The traditional international April mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan Tynysbek Sharipovich Kalmenov, pp. 169-170. 2021. (in Russian).

3. S.N. Kharin, T.A. Nauryz. The solution of two-phase Spherical Stefan problem by using linear combination of heat polynomials. Traditional International April math conference in honor Day of Science Workers of the Republic of Kazakhstan, dedicated to the 1150th anniversary of Abu Nasyr al-Farabi and 75th anniversary of the Institute of Mathematics and mathematical Modeling, p.125, 2020.

4. S.N. Kharin, T.A. Nauryz. Stefan problems with enhanced nonlinearity and its applications. Traditional International April math conference in honor Day of Science Workers of the Republic of Kazakhstan, pp. 16-19, 2022.

The structure and volume of the dissertation. The dissertation work consists of a title page, recognition, content, introduction, three sections and a conclusion, a list of references, an appendix. The total volume of the thesis is 110 pages, including 16 figures and 1 table.

The main content of the dissertation. The introduction reflects the justification of the relevance of the topic of the dissertation, purpose, object, subject, research objectives, justification of the scientific novelty of the work.

In the first chapter, we introduce the integral error function, heat polynomials and associated functions. Special functions for the generalized heat equation are presented such as the Laguerre polynomials and the degenerate hypergeometric function and the properties of the hypergeometric function are shown.

In the second chapter, the applications of heat polynomials and special functions are considered. First, one phase inverse spherical Stefan problem is considered where we have to find the temperatures of liquid and solid phases and the heat flux entering the electrical contact spots. Approximate solutions of heat fluxes are investigated through two approximating methods as a variational method and a collocation method. Comparing the numerical results, we find that the variational method will give us a very good approximation. In the second problem, we consider the solution of the two-phase Stefan problem for the generalized heat equation using special functions (Laguerre polynomial and degenerate hypergeometric function).

In the third chapter, we consider one phase spherical Stefan problem with coefficients depending on temperature. The method of similarity is investigated, which is very useful for simplifying the Stefan problem to an ordinary nonlinear differential equation of the second order with initial and boundary conditions and with Stefan conditions.